| Student Name: | BJUT ID: | UCD ID: |
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Institution: Beijing-Dublin International College
Module: University Physics 2 (BDIC2008J)
Lecturer: Dr. Hao Zhu

**Problem Set 2**

*Gauss' Law*

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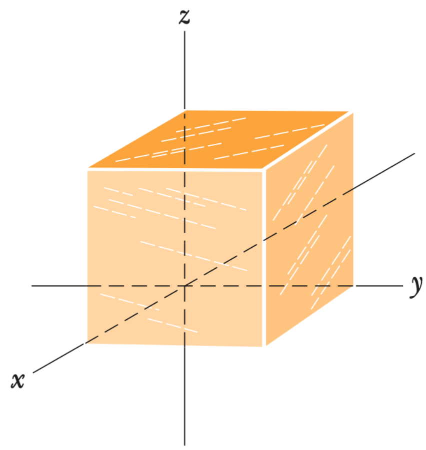
**Problem 1.** *The cube in the figure has edge length* 1.40m *and is oriented as shown in a region of uniform electric field. Find the electric flux through the right face if the electric field, in newtons per coulomb, is given by* ***(a)*** $6.00\vec{i}$, ***(b)*** $-2.00\vec{j}$, *and* ***(c)*** $-3.00\vec{i} + 4.00\vec{k}$. ***(d)*** *What is the total flux through the cube for each field?*




*Solution.* We use $\Phi = \vec{E} \cdot \vec{A}$, where $\vec{A} = A\vec{j} = (1.40\text{m})^2\vec{j}$.

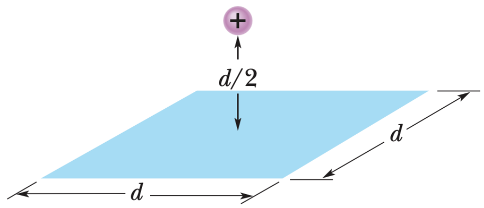
**(a)** $\Phi = (6.00\text{N/C})\vec{i} \cdot (1.40\text{m})^2\vec{j} = 0$.

**(b)** $\Phi = (-2.00\text{N/C})\vec{j} \cdot (1.40\text{m})^2\vec{j} = -3.92\text{N} \cdot \text{m}^2/\text{C}$.

**(c)** $\Phi = [(-3.00\text{N/C})\vec{i} + (4.00\text{N/C})\vec{k}] \cdot (1.40\text{m})^2\vec{j} = 0$.

**(d)** The total flux of a uniform field through a closed surface is always zero. □

**Problem 2.** *In the figure below, a proton is a distance $d/2$ directly above the centre of a square of side $d$. What is the magnitude of the electric flux through the square? (Hint: Think of the square as one face of a cube with edge $d$.)*




*Solution.* To exploit the symmetry of the situation, we imagine a closed Gaussian surface in the shape of a cube, of edge length $d$, with a proton of charge $q = +1.6 \times 10^{-19}\text{C}$ situated at the inside center of the cube. The cube has six faces, and we expect an equal amount of flux through each face. The total amount of flux is $\Phi_{net} = q/\varepsilon_0$, and we conclude that the flux through the square is one-sixth of that. Thus,

$$\Phi = \frac{q}{6\varepsilon_0} = \frac{1.6 \times 10^{-19}\text{C}}{6(8.85 \times 10^{-12}\text{C}^2/\text{N} \cdot \text{m}^2)} = 3.01 \times 10^{-9}\text{N} \cdot \text{m}^2/\text{C}$$

□

**Problem 3.** *The electric field in a certain region of Earth's atmosphere is directed vertically down. At an altitude of* 300m *the field has magnitude* 60.0N/C*; at an altitude of* 200m*, the magnitude is* 100N/C*. Find the net amount of charge contained in a cube* 100m *on edge, with horizontal faces at altitudes of* 200 *and* 300m*.*

*Solution.* A cube has six surfaces. The total flux through the cube is the sum of fluxes through each individual surface. We use Gauss' law to find the net charge inside the cube.

Let $A$ be the area of one face of the cube, $E_u$ be the magnitude of the electric field at the upper face, and $E_l$ be the magnitude of the field at the lower face. Since the field is downward, the flux through the upper face is negative and the flux through the lower face is positive. The flux through the other faces is zero (because their area vectors are parallel to the field), so the total flux through the cube surface is

$$\Phi = A(E_l - E_u)$$

The net charge inside the cube is given by Gauss' law: $q = \varepsilon_0 \Phi$. Substituting the values given, we find the net charge to be

$$\begin{aligned} q &= \varepsilon_0 \Phi = \varepsilon_0 A(E_l - E_u) = (8.85 \times 10^{-12} \mathrm{C}^2/\mathrm{N} \cdot \mathrm{m}^2)(100\mathrm{m})^2(100\mathrm{N/C} - 60.0\mathrm{N/C}) \\ &= 3.54 \times 10^{-6}\mathrm{C} = 3.54\mu\mathrm{C} \end{aligned}$$

Since $\Phi > 0$, we conclude that the cube encloses a net positive charge. $\square$

**Problem 4.** *Space vehicles travelling through Earth's radiation belts can intercept a significant number of electrons. The resulting charge buildup can damage electronic components and disrupt operations. Suppose a spherical metal satellite* 1.3m *in diameter accumulates* $2.4\mu$C *of charge in one orbital revolution.* ***(a)*** *Find the resulting surface charge density.* ***(b)*** *Calculate the magnitude of the electric field just outside the surface of the satellite, due to the surface charge.*

*Solution.* **(a)** The area of a sphere may be written $4\pi R^2 = \pi D^2$. Thus,

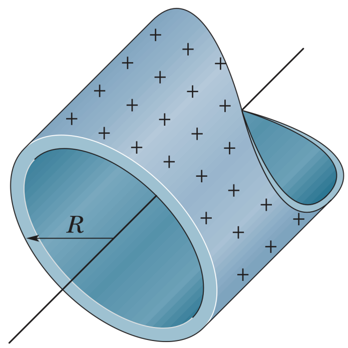
$$\sigma = \frac{q}{\pi D^2} = \frac{2.4 \times 10^{-6}\text{C}}{\pi(1.3\text{m})^2} = 4.5 \times 10^{-7}\text{C/m}^2$$

**(b)** For the conducting surface, the Gauss' Law becomes $EA = \dfrac{\sigma A}{\varepsilon_0} \Rightarrow E = \dfrac{\sigma}{\varepsilon_0}$, which gives

$$E = \frac{\sigma}{\varepsilon_0} = \frac{4.5 \times 10^{-7}\text{C/m}^2}{8.85 \times 10^{-12}\text{C}^2/\text{N} \cdot \text{m}^2} = 5.1 \times 10^4\text{N/C}$$

□

**Problem 5.** *This figure shows a section of a long, thin-walled metal tube of radius* $R = 3.00\text{cm}$, *with a charge per unit length of* $\lambda = 2.00 \times 10^{-8}\text{C/m}$. *What is the magnitude* $E$ *of the electric field at radial distance* ***(a)*** $r = R/2.00$ *and* ***(b)*** $r = 2.00R$? ***(c)*** *Graph* $E$ *versus* $r$ *for the range* $r = 0$ *to* $2.00R$.




*Solution.* We imagine a cylindrical Gaussian surface $A$ of radius $r$ and unit length concentric with the metal tube. Then by symmetry $\oint_A \vec{E} \cdot d\vec{A} = 2\pi r E = \frac{q_{encl}}{\varepsilon_0}$.

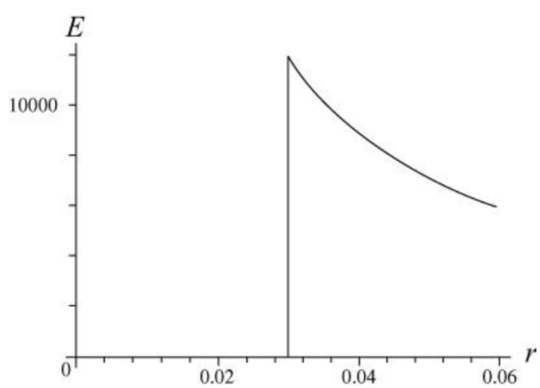
**(a)** For $r < R$, $q_{encl} = 0$, so $E = 0$.

**(b)** For $r > R$, $q_{encl} = \lambda$, so $E(r) = \lambda/2\pi r\varepsilon_0$. With $\lambda = 2.00 \times 10^{-8}\text{C/m}$ and $r = 2.00R = 0.06\text{m}$, we obtain

$$E = \frac{2.0 \times 10^{-8}\text{C/m}}{2\pi(0.06\text{m})(8.85 \times 10^{-12}\text{C}^2/\text{N} \cdot \text{m}^2)} = 5.99 \times 10^3\text{N/C}$$

**(c)** The plot of $E$ versus $r$ is shown to the below. Here, the maximum value is

$$E_{max} = \frac{\lambda}{2\pi r\varepsilon_0} = \frac{2.0 \times 10^{-8}\text{C/m}}{2\pi(0.03\text{m})(8.85 \times 10^{-12}\text{C}^2/\text{N} \cdot \text{m}^2)} = 1.2 \times 10^4\text{N/C}$$




□

**Problem 6.** *A long, straight wire has fixed negative charge with a linear charge density of magnitude* $3.6\text{nC/m}$*. The wire is to be enclosed by a coaxial, thin-walled nonconducting cylindrical shell of radius* $1.5\text{cm}$*. The shell is to have positive charge on its outside surface with a surface charge density* $\sigma$ *that makes the net external electric field zero. Calculate* $\sigma$*.*

*Solution.* We denote the radius of the thin cylinder as $R = 0.015\text{m}$. Using $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$ (line of charge), the net electric field for $r > R$ is given by

$$E_{net} = E_{wire} + E_{cylinder} = \frac{-\lambda}{2\pi\varepsilon_0 r} + \frac{\lambda'}{2\pi\varepsilon_0 r}$$

where $-\lambda = -3.6\text{nC/m}$ is the linear charge density of the wire and $\lambda'$ is the linear charge density of the thin cylinder. We note that the surface and linear charge densities of the thin cylinder are related by

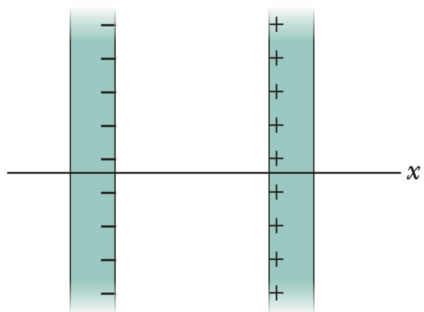
$$q_{cylinder} = \lambda' L = \sigma(2\pi RL) \Rightarrow \lambda' = \sigma(2\pi R)$$

Now, $E_{net}$ outside the cylinder will equal zero, provided that $2\pi R\sigma = \lambda$, or

$$\sigma = \frac{\lambda}{2\pi R} = \frac{3.6 \times 10^{-6}\text{C/m}}{(2\pi)(0.015\text{m})} = 3.8 \times 10^{-8}\text{C/m}^2$$

□

**Problem 7.** *In the figure below, two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have excess surface charge densities of opposite signs and magnitude* $7.00 \times 10^{-22}\mathrm{C/m^2}$*. In unit-vector notation, what is the electric field at points* ***(a)*** *to the left of the plates,* ***(b)*** *to the right of them, and* ***(c)*** *between them?*




*Solution.* We use the formula $E = \dfrac{\sigma}{2\varepsilon_0}$ for the sheet of charge.

**(a)** To the left of the plates:

$$\vec{E} = \left(\frac{\sigma}{2\varepsilon_0}\right)(-\vec{\imath}) \text{ (from the right plate)} + \left(\frac{\sigma}{2\varepsilon_0}\right)\vec{\imath} \text{ (from the left one)} = 0$$

**(b)** To the right of the plates:

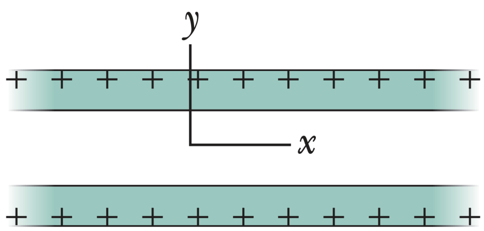
$$\vec{E} = \left(\frac{\sigma}{2\varepsilon_0}\right)\vec{\imath} \text{ (from the right plate)} + \left(\frac{\sigma}{2\varepsilon_0}\right)(-\vec{\imath}) \text{ (from the left one)} = 0$$

**(c)** Between the plates:

$$\vec{E} = \left(\frac{\sigma}{2\varepsilon_0}\right)(-\vec{\imath}) + \left(\frac{\sigma}{2\varepsilon_0}\right)(-\vec{\imath}) = \left(\frac{\sigma}{\varepsilon_0}\right)(-\vec{\imath}) = -\left(\frac{7.00 \times 10^{-22}\mathrm{C/m^2}}{8.85 \times 10^{-12}\mathrm{C^2/N \cdot m^2}}\right)\vec{\imath}$$
$$= (-7.91 \times 10^{-11}\mathrm{N/C})\vec{\imath}$$

□

**Problem 8.** *The Figure below shows cross sections through two large, parallel, non-conducting sheets with identical distributions of positive charge with surface charge density* $s = 1.77 \times 10^{-22}\text{C/m}^2$*. In unit-vector notation, what is* $\vec{E}$ *at points* ***(a)*** *above the sheets,* ***(b)*** *between them, and* ***(c)*** *below them?*



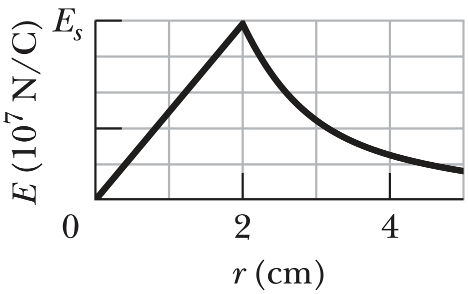

*Solution.* The electric field due to either sheet of charge with surface charge density $\sigma = 1.77 \times 10^{-22}\text{C/m}^2$ is perpendicular to the plane of the sheet (pointing *away* from the sheet if the charge is positive) and has magnitude $E = \sigma/2\varepsilon_0$. Using the superposition principle, we conclude:

**(a)** $E = \sigma/\varepsilon_0 = (1.77 \times 10^{-22}\text{C/m}^2)/(8.85 \times 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2) = 2.00 \times 10^{-11}\text{N/C}$, pointing in the upward direction, or $\vec{E} = (2.00 \times 10^{-11}\text{N/C})\vec{j}$;

**(b)** $E = 0$;

**(c)** and, $E = \sigma/\varepsilon_0$, pointing down, or $\vec{E} = -(2.00 \times 10^{-11}\text{N/C})\vec{j}$. □

**Problem 9.** *The figure below gives the magnitude of the electric field inside and outside a sphere with a positive charge distributed uniformly throughout its volume. The scale of the vertical axis is set by* $E_s = 5.0 \times 10^7 \mathrm{N/C}$. *What is the charge on the sphere?*

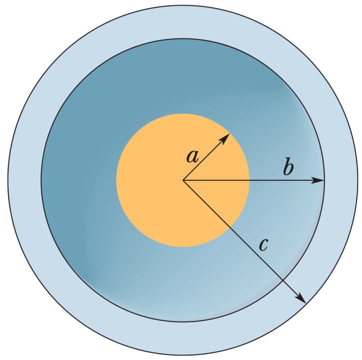



*Solution.* We determine the (total) charge on the ball by examining the maximum value ($E = 5.0 \times 10^7 \mathrm{N/C}$) shown in the graph (which occurs at $r = 0.020\mathrm{m}$). Thus, from $E = q/4\pi\varepsilon_0 r^2$, we obtain

$$q = 4\pi\varepsilon_0 r^2 E = \frac{(0.020\mathrm{m})^2(5.0 \times 10^7\mathrm{N/C})}{8.99 \times 10^9 \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2} = 2.2 \times 10^{-6}\mathrm{C}$$

$\square$

**Problem 10.** *The solid sphere of radius* $a = 2.00\text{cm}$ *is concentric with a spherical conducting shell of inner radius* $b = 2.00a$ *and outer radius* $c = 2.40a$*. The sphere has a net uniform charge* $q_1 = +5.00\text{fC}$*; the shell has a net charge* $q_2 = -q_1$*. What is the magnitude of the electric field at radial distances* ***(a)*** $r = 0$*,* ***(b)*** $r = a/2.00$*,* ***(c)*** $r = a$*,* ***(d)*** $r = 1.50a$*,* ***(e)*** $r = 2.30a$*, and* ***(f)*** $r = 3.50a$*? What is the net charge on the* ***(g)*** *inner and* ***(h)*** *outer surface of the shell?*




*Solution.* At all points where there is an electric field, it is radially outward. For each part of the problem, use a Gaussian surface in the form of a sphere that is concentric with the sphere of charge and passes through the point where the electric field is to be found. The field is uniform on the surface, so $\oint_A \vec{E} \cdot d\vec{A} = 4\pi r^2 E$, where $r$ is the radius of the Gaussian surface. For $r < a$, the charge enclosed by the Gaussian surface is $q_1(r/a)^3$. Gauss' law yields

$$4\pi r^2 E = \left(\frac{q_1}{\varepsilon_0}\right)\left(\frac{r}{a}\right)^3 \Rightarrow E = \frac{q_1 r}{4\pi\varepsilon_0 a^3}$$

**(a)** For $r = 0$, the above equation implies $E = 0$.

**(b)** For $r = a/2$, we have

$$E = \frac{q_1(a/2)}{4\pi\varepsilon_0 a^3} = \frac{(8.99 \times 10^9\text{N} \cdot \text{m}^2/\text{C}^2)(5.00 \times 10^{-15}\text{C})}{2(2.00 \times 10^{-2}\text{m})^2} = 5.62 \times 10^{-2}\text{N/C}$$

**(c)** For $r = a$, we have

$$E = \frac{q_1}{4\pi\varepsilon_0 a^2} = \frac{(8.99 \times 10^9\text{N} \cdot \text{m}^2/\text{C}^2)(5.00 \times 10^{-15}\text{C})}{(2.00 \times 10^{-2}\text{m})^2} = 0.112\text{N/C}$$

**(d)** For $r = 1.50a$, we have

$$E = \frac{q_1}{4\pi\varepsilon_0 r^2} = \frac{(8.99 \times 10^9\text{N} \cdot \text{m}^2/\text{C}^2)(5.00 \times 10^{-15}\text{C})}{(1.50 \times 2.00 \times 10^{-2}\text{m})^2} = 0.0499\text{N/C}$$

**(e)** In the region $b < r < c$, since the shell is conducting, the electric field is zero. Thus, for

$r = 2.30a$, we have $E = 0$.

**(f)** For $r > c$, the charge enclosed by the Gaussian surface is zero. Gauss' law yields $4\pi r^2 E = 0 \Rightarrow E = 0$. Thus, $E = 0$ at $r = 3.50a$.

**(g)** Consider a Gaussian surface that lies completely within the conducting shell. Since the electric field is everywhere zero on the surface, $\oint \vec{E} \cdot d\vec{A} = 0$ and, according to Gauss' law, the net charge enclosed by the surface is zero. If $Q_i$ is the charge on the inner surface of the shell, then $q_1 + Q_i = 0$ and $Q_i = -q_1 = -5.00\text{fC}$.

**(h)** Let $Q_o$ be the charge on the outer surface of the shell. Since the net charge on the shell is $-q$, $Q_i + Q_o = -q_1$. This means

$$Q_o = -q_1 - Q_i = -q_1 - (-q_1) = 0$$

□